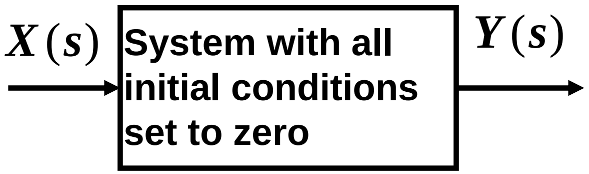
Transfer Function

Transfer Function



$$H(s) = \frac{Y(s)}{X(s)}$$

If the model for the system is a differential equation

$$b_n \frac{d^n y}{dt^n} + b_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \dots + b_1 \frac{dy}{dt} + b_o y$$
$$= a_m \frac{d^m x}{dt^m} + a_{m-1} \frac{d^{m-1} x}{dt^{m-1}} + \dots + a_1 \frac{dx}{dt} + a_o x$$

If all initial conditions are zero

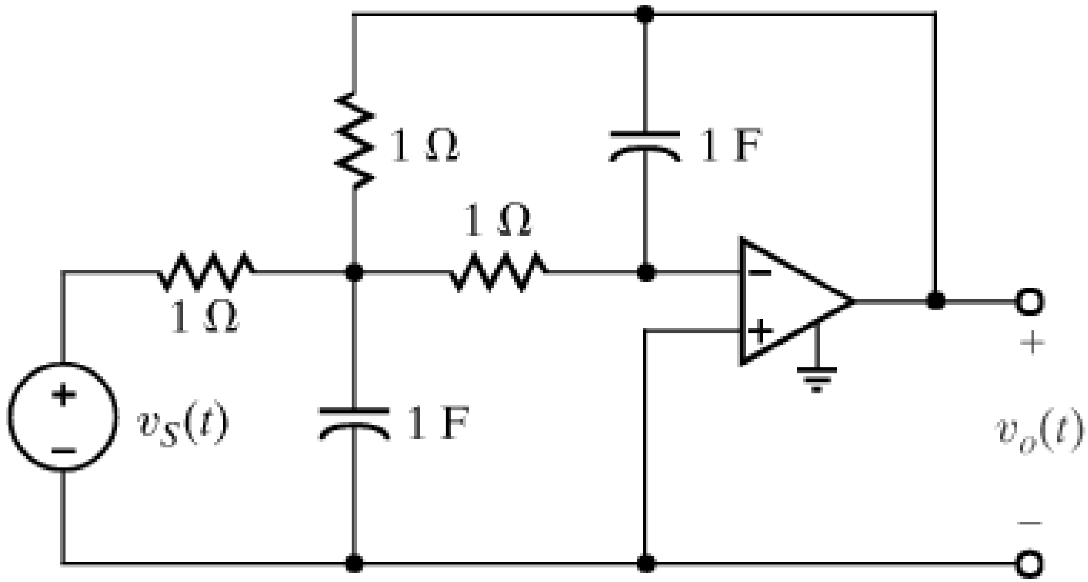
$$\mathcal{L}\left[\frac{d^k y}{dt^k} \right] = s^k Y(s)$$

$$b_n s^n Y(s) + \dots + b_1 s Y(s) + b_0 Y(s)$$
$$= a_m s^m X(s) + \dots + a_1 s X(s) + a_0 X(s)$$

$$Y(s) = \frac{b_n s^n + \dots + b_1 s + b_0}{a_m s^m + \dots + a_1 s + a_0} X(s)$$

$$H(s) = \frac{b_n s^n + \dots + b_1 s + b_0}{a_m s^m + \dots + a_1 s + a_0}$$

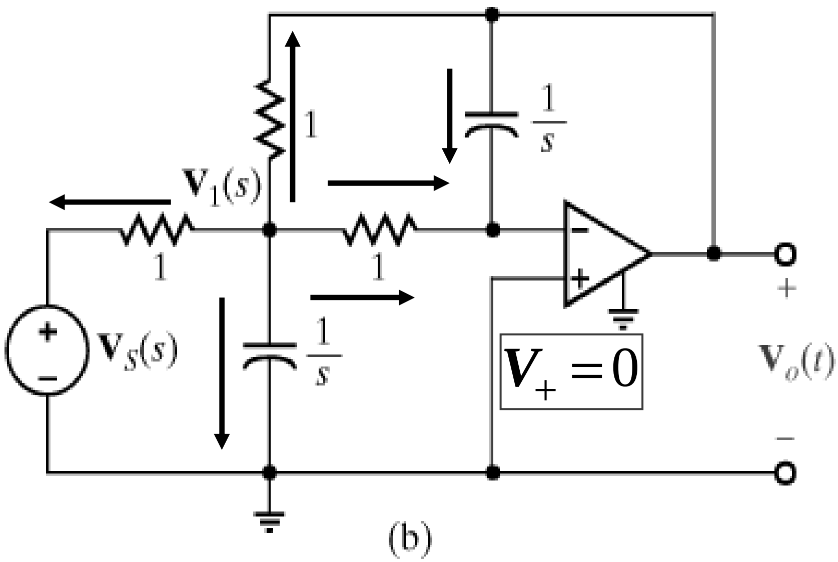
EXAMPLE



(a)

Determine the transfer function and the type of damping

Solution



$$V_+ = 0$$

$$\frac{V_1(s) - V_S(s)}{1} + \frac{V_1(s)}{1/s} + \frac{V_1(s)}{1} + \frac{V_1(s) - V_0(s)}{1} = 0$$

$$V_1(s) = -sV_o(s) \Leftarrow \frac{V_1(s)}{1} + \frac{V_o(s)}{1/s} = 0$$

$$\frac{V_o(s)}{V_S(s)} = \frac{1}{s^2 + 3s + 1}$$

$$\omega_o^2 \Rightarrow \omega_0 = 1$$

$$2\varsigma\omega_o \Rightarrow \varsigma = \frac{3}{2}$$